Sequences in London 2024

Sensitivity of string compressors and repetitiveness measures

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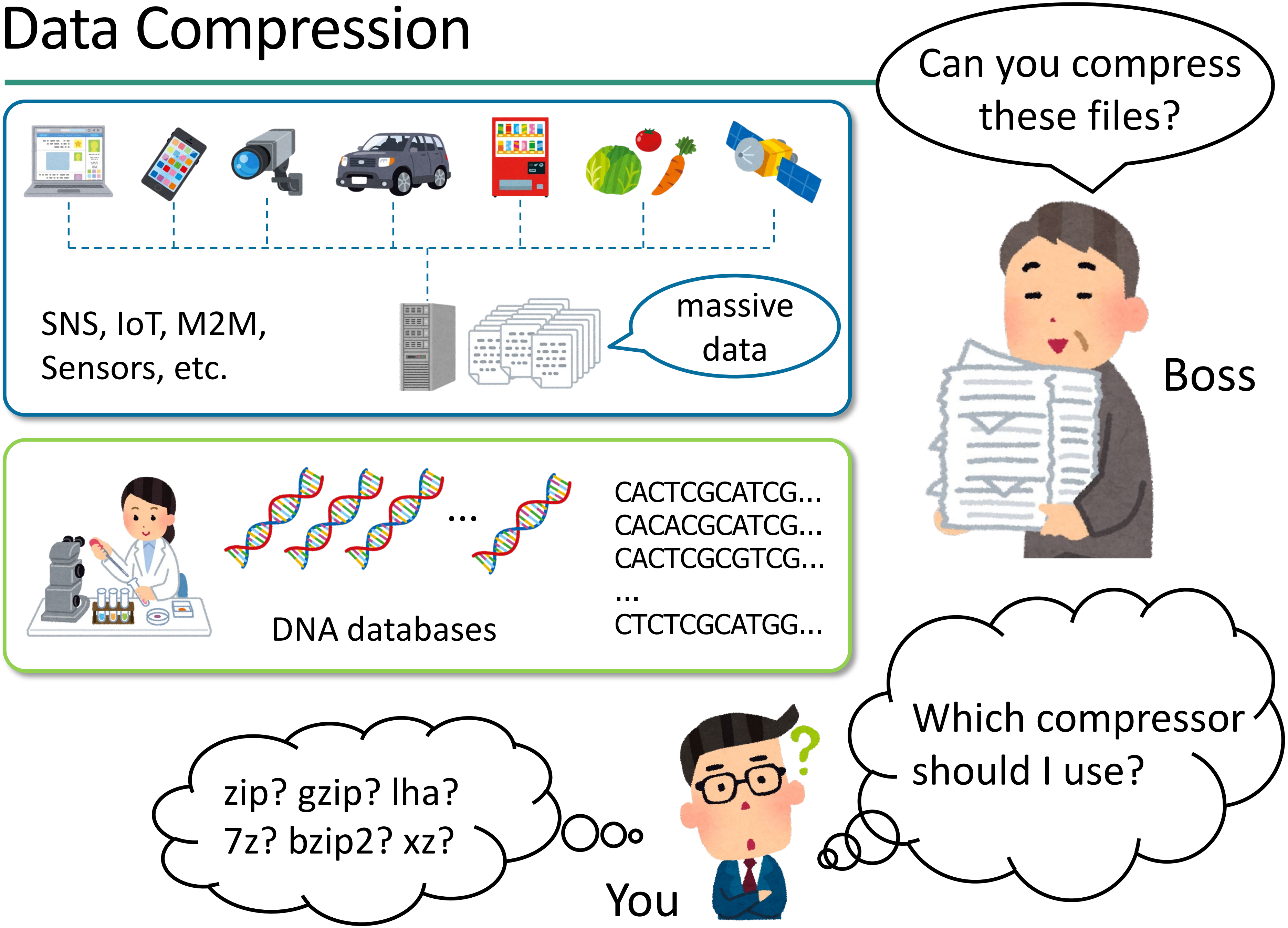
Data Compression



New Criterion for Compressors

Review 1 for Compressor C

T = ababaaba...ab

$C(T)$

(1) Excellent compression ratio!
(2) Super fast (de)compression!
(3) Searchable compression!

Review 2 for Compressor C

T' = a**X**abaaba..ab

$C(T')$

But the compressed size blew up just after a small edit operation!!

The compressor C looks great because it satisfies all the known criteria (1),(2),(3)!

What!? Can it happen? My company deals with dynamic data..., so it can be problematic.

New criterion **(4) Compression Sensitivity**

Compression Sensitivity

[Definition]

Sensitivity of compressor C

T' is any string that can be obtained by performing a single character edit operation on T.

Worst-case multiplicative sensitivity

$$\max\{|C(T')|/|C(T)| : T \in \Sigma^n, \mathrm{EditDistance}(T, T') = 1\}$$

Worst-case additive sensitivity

$$\max\{|C(T')| - |C(T)| : T \in \Sigma^n, \mathrm{EditDistance}(T, T') = 1\}$$

[Intuition]

Small sensitivity $\Longleftrightarrow$ Robust compression under edits/errors

Multiplicative Compression Sensitivity

Compressor / Measure	Upper Bound	Lower Bound
LZ77 z		
LZSS s		
Bidirectional scheme b		
Smallest grammar g^*		
BISECTION g_{BISCTN}		
GCIS g_{GCIS}		
LZ78 g_{78}		$\Omega(n^{1/4})$ [Lagarde & Perifel SODA 2018]
RLBWT r		$\Omega(\log n)$ [SOFSEM 2021]
Substring complexity δ		

Very few previous work

Lower Bound for RLBWT r

r: RLE size of the rightmost column of the lexicographically sorted rotations of the input string.

Reversed Fibonacci word s_i^{rev}

	s_6^{rev} = baabaababaabb	BWT
2	aabaababaabab	b
10	aababaabaabab	b
5	aababaababaab	b
13	abaabaababaab	b
8	abaababaabaab	b
3	abaababaababa	a
11	ababaabaababa	a
6	ababaababaaba	a
1	baabaababaaba	a
9	baababaabaaba	a
4	baababaababaa	a
12	babaabaababaa	a
7	babaababaabaa	a

	$(s_6\mathrm{b})^{\mathrm{rev}}$ = bbaabaababaabb	BWT
3	aabaababaababb	b
6	aababaababbaab	b
11	aababbaabaabab	b
4	abaababaababba	a
9	abaababbaabaab	b
7	ababaababbaaba	a
12	ababbaabaababa	a
14	abbaabaababaab	b
2	baabaababaabab	b
5	baababaababbaa	a
10	baababbaabaaba	a
8	babaababbaabaa	a
13	babbaabaababaa	a
1	bbaabaababaaba	a

$i = \log_\phi n$

$r(s_i^{\mathrm{rev}}) = 2$

$r(\mathrm{b}s_i^{\mathrm{rev}}) = \Omega(\log n)$

Multi. Sensitivity of Repetitiveness Measures & Dictionary Comp.

Compressor / Measure	Edit Operation	Upper Bound	Lower Bound
Substring complexity δ	del	1.5	1.5
	ins/sub	2	2
Smallest string attractor γ	any	$O(\log n)$	2.5 [Bannai et al. ESA 2022]
Bidirectional scheme b	ins/sub	2	2
	del	2	1.5
LZ77 z	any	2	2
LZSS s	ins	2	2
	del/sub	3	3
LZ78 g_{78}	ins	$O((n / \log n)^{2/3})$	$\Omega(n^{1/4})$ [Lagarde & Perifel SODA 2018]
	del/sub		$\Omega(n^{1/4})$
LZEnd z_{End}	any	$O(\log^2 (n / \mathrm{d}))$	2
RLBWT r	any	$O(\log r \log n)$	$\Omega(\log n)$

Multiplicative Sensitivity of Grammar Compressors

Grammar compressors	Edit Operation	Upper Bound	Lower Bound
Smallest grammar g^*	any	2	-
α-balanced	any	$O(\log(n / g^*))$	-
AVL-grammar			
RePair	any	$O((n / \log n)^{2/3})$	-
Longest-Match			
Greedy			
Bisection	sub	2	2
	ins/del	$\lvert\Sigma\rvert+1$	$\lvert\Sigma\rvert$
GCIS	any	4	4
CDAWG	sub	8	2
	ins/del	13	2

Multi. Sensitivity of Repetitiveness Measures & Dictionary Comp.

Compressor / Measure	Edit Operation	Upper Bound	Lowe Bound
Substring complexity δ	del	1.5	1.5
	ins/sub	2	2
Smallest string attractor γ	any	$O(\log n)$	2
Bidirectional scheme b	ins/sub		[Bannai et al. A 2022]
	del		
LZ77 z	any		2
LZSS s	ins		2
	del/sub	3	3
LZ78 g_{78}	ins	$O((n / \log n)^{2/3})$	$\Omega(n^{1/4})$ [Lagarde & Perifel SODA 2018]
	del/sub		$\Omega(n^{1/4})$
LZEnd z_{End}	any	$O(\log^2 (n / \mathrm{d}))$	2
RLBWT r	any	$O(\log r \log n)$	$\Omega(\log n)$

I will talk about this

LZSS (zip / gzip / lha / 7zip / png ...)

LZSS [Storer-Szymanski, 1982]

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
$T =$	a	b	a	a	b	a	b	a	b	a	b	a	b	a	b

Scans T from left to right and parses T

- at each fresh letter, or
- at the ending position of the longest previously occurring substring.

LZSS (zip / gzip / lha / 7zip / png ...)

LZSS [Storer-Szymanski, 1982]

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
$T =$	a |	b	a	a	b	a	b	a	b	a	b	a	b	a	b

Fresh letter

Scans T from left to right and parses T

- at each fresh letter, or
- at the ending position of the longest previously occurring substring.

LZSS (zip / gzip / lha / 7zip / png ...)

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	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
$T =$	a |	b |	a	a	b	a	b	a	b	a	b	a	b	a	b

Fresh letter

Scans T from left to right and parses T

- at each fresh letter, or
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LZSS (zip / gzip / lha / 7zip / png ...)

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	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
$T =$	a	b	a	a	b	a	b	a	b	a	b	a	b	a	b

The ending position of the longest previously occurring substring.

Scans T from left to right and parses T

- at each fresh character, or
- at the ending position of the longest previously occurring substring.

LZSS (zip / gzip / lha / 7zip / png ...)

LZSS [Storer-Szymanski, 1982]

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
$T =$	a	b	a	a	b	a	b	a	b	a	b	a	b	a	b

The ending position of the longest previously occurring substring.

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$T =$	a	b	a	a	b	a	b	a	b	a	b	a	b	a	b

The ending position of the longest previously occurring substring.

LZSS (zip / gzip / lha / 7zip / png ...)

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	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
$T =$	a	b	a	a	b	a	b	a	b	a	b	a	b	a	b

The ending position of the longest previously occurring substring.

LZSS (zip / gzip / lha / 7zip / png ...)

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	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
$T =$	a|	b|	a|	a	b	a|	b	a|	b	a	b	a|	b	a	b|

The ending position of the longest previously occurring substring.

LZSS (zip / gzip / lha / 7zip / png ...)

LZSS [Storer-Szymanski, 1982]

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
$T =$	a |	b |	a |	a	b	a |	b	a |	b	a	b	a |	b	a	b |

(5, 7)

LZSS (zip / gzip / lha / 7zip / png ...)

LZSS [Storer-Szymanski, 1982]

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

$$T = \mathrm{a} \mid \mathrm{b} \mid \mathrm{a} \mid \mathrm{a}\ \mathrm{b}\ \mathrm{a} \mid \mathrm{b}\ \mathrm{a} \mid \mathrm{b}\ \mathrm{a}\ \mathrm{b}\ \mathrm{a} \mid \mathrm{b}\ \mathrm{a}\ \mathrm{b} \mid$$

(a) (b) (1, 1) (1, 3) (2, 3) (5, 8) (5, 7)

Compression size (# of phrases) $= 7$

LZSS (zip / gzip / lha / 7zip / png ...)

LZSS [Storer-Szymanski, 1982]

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
$T =$	a|	b|	a|	a	b	a|	b	a|	b	a	b	a|	b	a	b|

(a) (b) (1, 1) (1, 3) (2, 3) (5, 8) (5, 7)

Compression size (# of phrases) $= 7$

LZSS with self-references

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
$T =$	a|	b|	a|	a	b	a|	b	a	b	a	b	a	b	a	b|

(a) (b) (1, 1) (1, 3) (5, 13)

Compression size (# of phrases) $= 5$

Upper Bound for Sensitivity of LZSS

s : LZSS compression size before edit
s': LZSS compression size after edit

Theorem

For any string, we have the following:

Multiplicative sensitivity for LZSS is $s'/s \leq 3.$

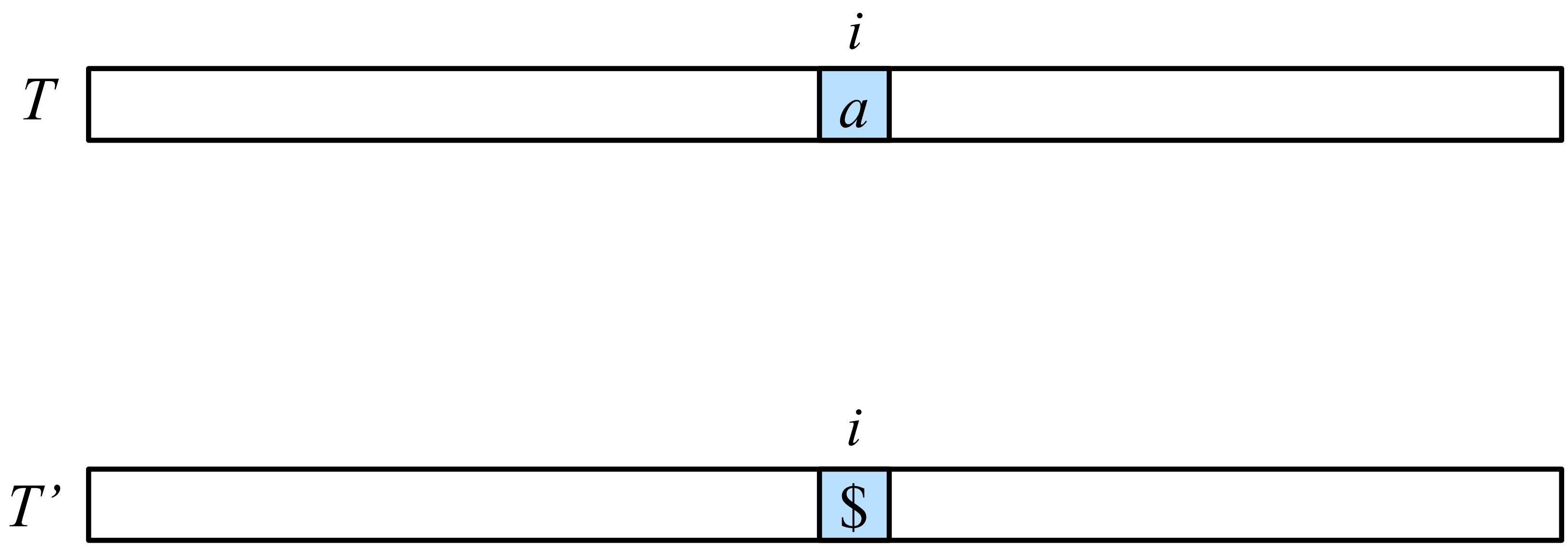
Additive sensitivity for LZSS is $s' - s \leq 2s - 2.$

These upper bounds hold for both versions of LZSS with/without self-references.

Upper Bound for Sensitivity of LZSS

Proof for $s' \leq 3s$ (without self-references)

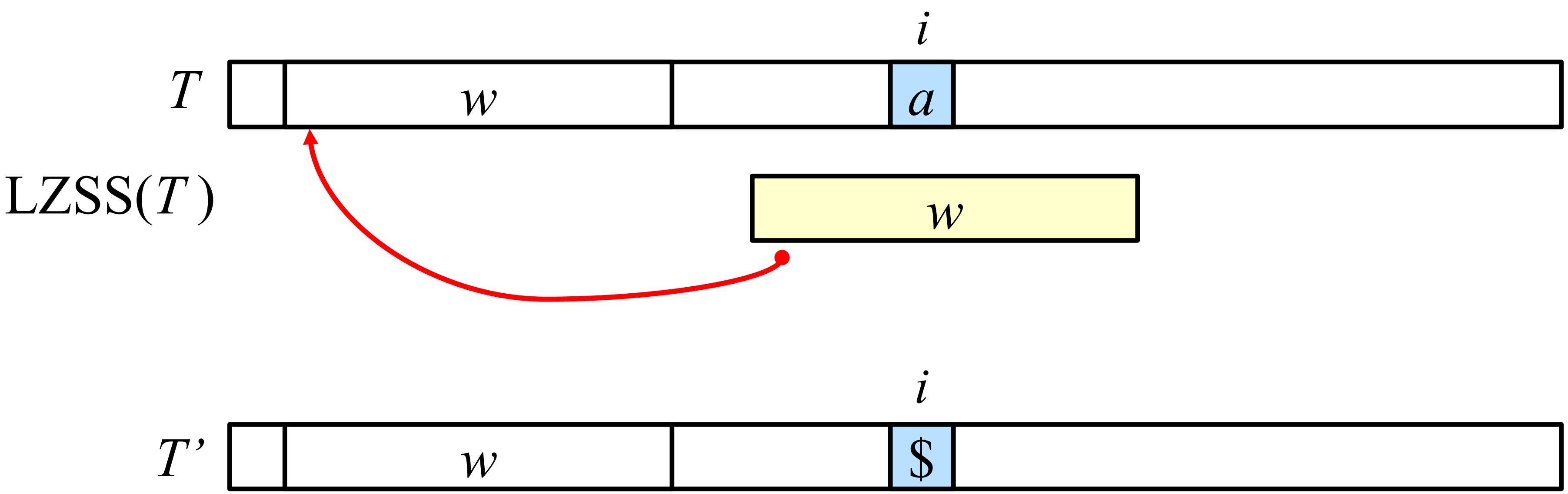
Claim 1: For the phrase w that contains the edited position i, # phrases starting in the interval for w can increase to at most 3.



Upper Bound for Sensitivity of LZSS

Proof for $s' \leq 3s$ (without self-references)

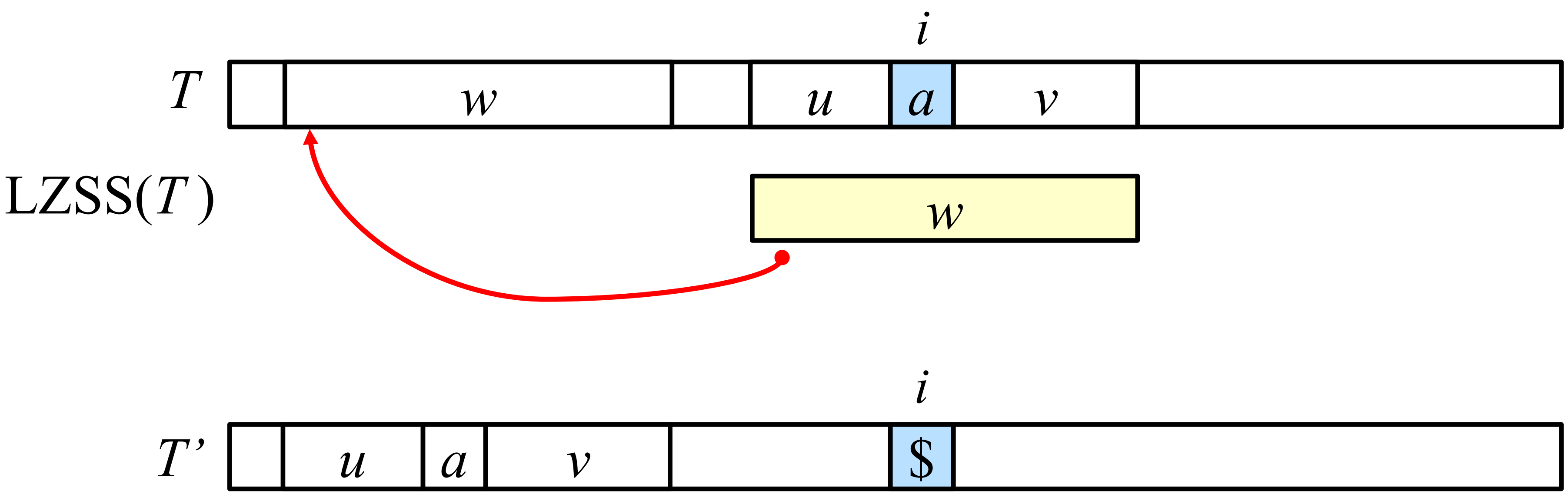
Claim 1: For the phrase w that contains the edited position i, # phrases starting in the interval for w can increase to at most 3.



Upper Bound for Sensitivity of LZSS

Proof for $s' \leq 3s$ (without self-references)

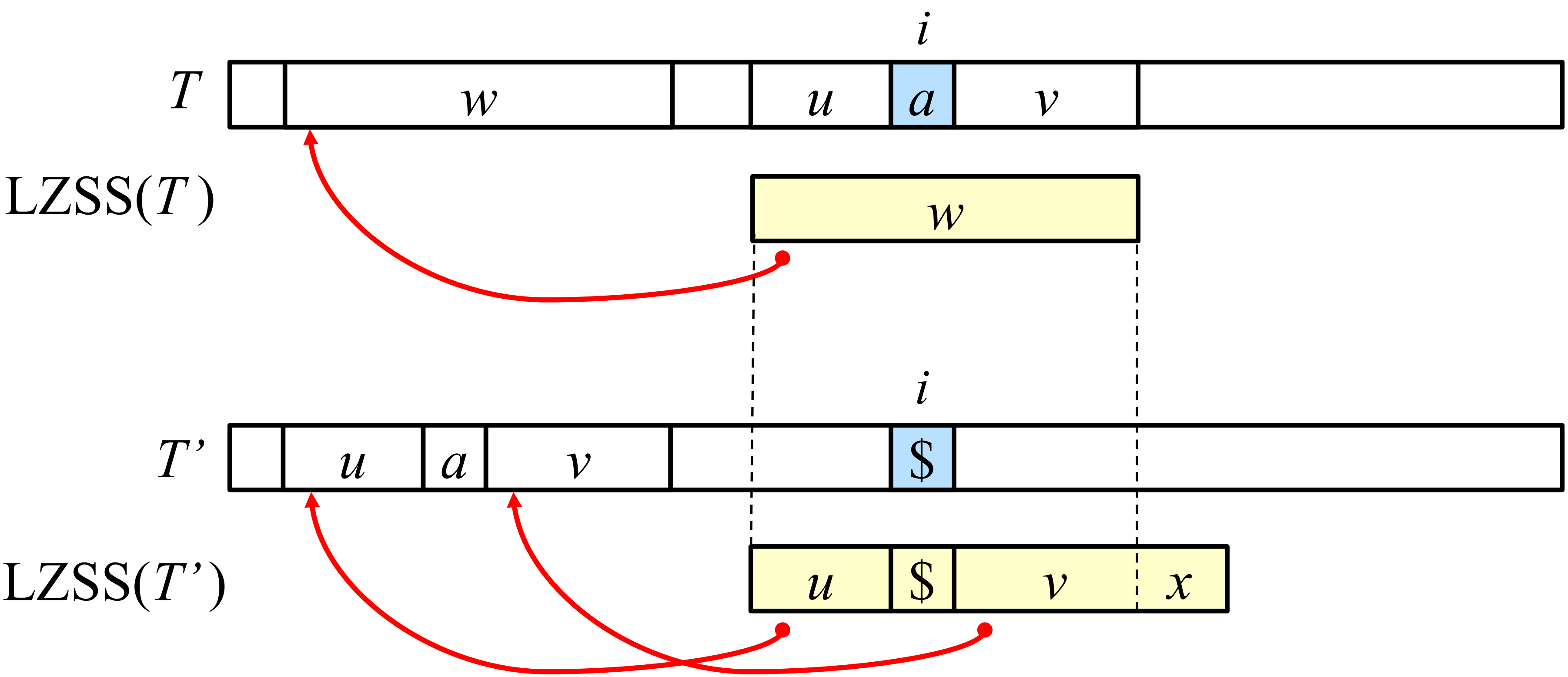
Claim 1: For the phrase w that contains the edited position i, # phrases starting in the interval for w can increase to at most 3.



Upper Bound for Sensitivity of LZSS

Proof for $s' \leq 3s$ (without self-references)

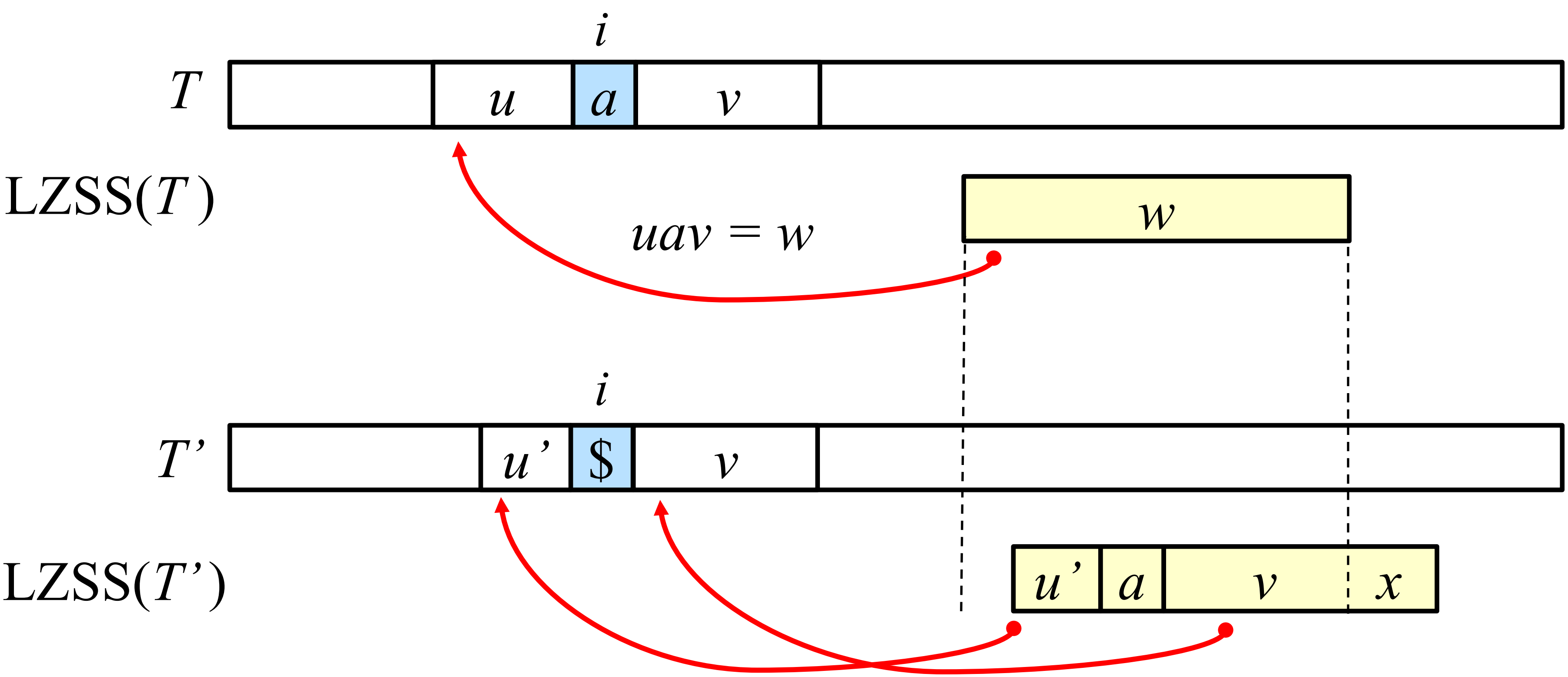
Claim 1: For the phrase w that contains the edited position i, # phrases starting in the interval for w can increase to at most 3.



Upper Bound for Sensitivity of LZSS

Proof for $s' \leq 3s$ (without self-references)

Claim 2: For any phrase w to the right of the edited position i, # phrases starting in the interval for w can increase to at most 3.



Upper/Lower Bounds for Sensitivity of LZSS

Theorem (Upper Bound)

For any string, we have the following:

Multiplicative sensitivity for LZSS is $s'/s \leq 3$.

Additive sensitivity for LZSS is $s' - s \leq 2s - 2$.

Theorem (Lower Bound)

There exists a family of strings of length n such that:

Multiplicative sensitivity for LZSS is $\lim_{s \to \infty} \inf(s'/s) = 3$.

Additive sensitivity for LZSS is $s' - s = 2s - \Theta(\sqrt{s}) = \Omega(\sqrt{n})$.

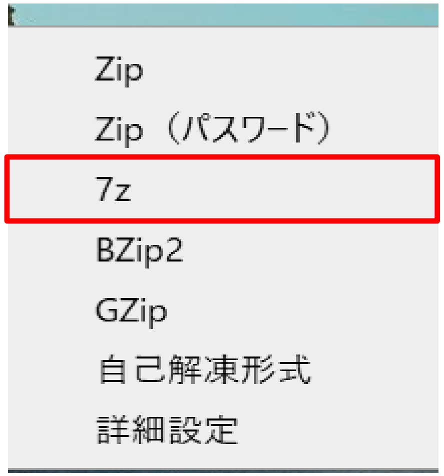
For the lower bound, we showed strings that have the aforementioned worst-case situations for most of their phrases.

Sensitivity of Compression Software

Does similar happen for
a real-world compression software?

We tested compression software

- 7zip = LZSS + RangeCoder.
- Supports **.7z** compression format.
- Compression ratio of 7zip is usually better than that of zip by 30-70%.



Sensitivity of Compression Software (7zip)

stringX： length 1,172,887 (1.12 MB)

!“#$%&‘()*+,./0123456789:;⇔?@ABCDEFGHIJKLMN!”#$%&’()*+,./01234
56789:;⇔?@ABCDEFGHIJKLM!“#$%&‘()*+,./0123456789:;<=>?@ABCDEF
GHIJKL!”#$%&’()*+,./0123456789:;<=>?@ ABCDE … ’!“#$%&!
”#$%!“#$!”#!“!!**{**OOPOPQOPQROPQRSOPQRSTOPQRSTUOPQRST …

Replace the 1083rd letter “ **{** “ with “ **~** ”

stringXp

!“#$%&‘()*+,./0123456789:;⇔?@ABCDEFGHIJKLMN!”#$%&’()*+,./01234
56789:;⇔?@ABCDEFGHIJKLM!“#$%&‘()*+,./0123456789:;<=>?@ABCDEF
GHIJKL!”#$%&’()*+,./0123456789:;<=>?@ ABCDE … ’!“#$%&!
”#$%!“#$!”#!“!!**~**OOPOPQOPQROPQRSOPQRSTOPQRSTUOPQRST …

7zip compressed size	stringX.7z	4,884 byte
	stgingXp.7z	7,189 byte

1.5 times bigger!

References

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